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**AN ANALYSIS OF THE FACTORS THAT AFFECT THE EXHAUST PROCESS
OF A FOUR-STROKE-CYCLE RECIPROCATING ENGINE**

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AN ANALYSIS OF THE FACTORS THAT AFFECT THE EXHAUST PROCESS
OF A FOUR-STROKE-CYCLE RECIPROCATING ENGINE

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SUMMARY

An analysis of the factors affecting the exhaust process of a four-stroke-cycle reciprocating engine was made by investigating the effect of changes in these factors upon the calculated cylinder pressure near the end of the exhaust period. If the effects of the exhaust pipe can be neglected and the variation in exhaust-valve flow coefficient with engine crank angle can be approximated by a sine curve, the significant factors affecting the exhaust process are found to be the gas-velocity parameter (which includes the average piston speed, the piston area, the speed of sound in the gases at the effective exhaust-valve opening angle, the exhaust-valve flow coefficient at the maximum valve lift, and the nominal exhaust-valve area), the initial gas-pressure ratio (which is the ratio of the cylinder pressure at the effective exhaust-valve opening angle to the exhaust pressure), and the effective exhaust-valve closing angle. The factors that have an insignificant effect upon the exhaust process were found to be the effective exhaust-valve opening angle, the compression ratio, and the crank throw to connecting-rod length ratio.

The analysis further showed that for most of the practical operating conditions, no optimum ratio of intake- to exhaust-valve area exists and the intake-manifold pressure does not have a significant effect upon the exhaust process.

A maximum value of the gas-velocity parameter equal to 0.325 is recommended. From this value the minimum exhaust-valve area can be calculated for any given piston speed and piston area.

From the analysis it is concluded that closing the exhaust valve late is the most effective method of keeping cylinder pressure low near the end of the exhaust stroke, as the value of the gas-velocity parameter is increased.

The results of this analysis can be used to estimate the cylinder pressures at 20° B.T.C. during the exhaust stroke of any engine. The magnitude of this estimated cylinder pressure is indicative of the adequacy of the exhaust-valve size and timing; high values of this pressure indicate the need for larger exhaust valves or different valve timing.

INTRODUCTION

An investigation was made to determine the relative significance of the factors that affect the exhaust process; the effects that an exhaust-process change has on cylinder charging were given special attention. Such a detailed study has not been reported elsewhere although a number of investigators have experimented with one or another of the various factors, which are the gas-velocity parameter, the initial gas-pressure ratio, and the effective exhaust-valve closing angle. An analysis of these factors requires the determination of their effect on the cylinder pressures near the end of the exhaust stroke. These pressures can be calculated by means of a differential equation that relates the piston motion to the flow of gases through the exhaust valve. Differential equations of this nature have been developed by Kemble (reference 1) but the forms of these equations are such that general conclusions regarding the various factors affecting the exhaust process cannot be determined.

A generalized form of the differential equation is presented, which was so developed that the calculated results are applicable to most four-stroke-cycle reciprocating engines and may be used to arrive at general conclusions regarding the relative significance of the various factors affecting the exhaust process. The range of factors investigated is sufficient to cover any practical case that might arise and special effort has been made to include the low initial gas-pressure ratios that correspond to the high back pressures currently used by reciprocating engines in combination with exhaust-gas turbines.

SYMBOLS

The following symbols are used in the analysis:

- A_e nominal exhaust-valve area, (sq in.)
 A_p piston area, (sq in.)

C	exhaust-valve flow coefficient at angle θ
C_a	average flow coefficient of exhaust valve for sine-wave valve-lift curve approximation
C'_a	average flow coefficient of exhaust valve for actual valve-lift curve
C_{max}	exhaust-valve flow coefficient for maximum valve lift
c_0	speed of sound at T_0 , (ft/sec)
d	nominal valve diameter, (in.)
f_1	$\frac{1}{r - 1}$
f_2	$\frac{V - V_{cl}}{V_D}$
f_3	$\frac{(dv/d\theta)}{V_D}$
g	acceleration of gravity, (ft/sec ²)
L	valve lift at crank angle θ , (in.)
l	connecting-rod length, (in.)
m	gas mass, (lb)
N	engine speed, (rpm)
P	cylinder pressure at crank angle θ , (lb/sq in.)
P_e	exhaust pressure, (lb/sq in.)
P/P_e	gas-pressure ratio
P_m	intake-manifold pressure, (lb/sq in.)
P_0	cylinder pressure at crank angle θ_0 , (lb/sq in.)
P_{340}	cylinder pressure at 340° A.T.C., (lb/sq in.)
R	gas constant, (ft-lb/lb/°F absolute)

r	compression ratio
S	average piston speed, (ft/min)
s	crank throw, (in.)
T	gas temperature, ($^{\circ}\text{R}$)
T_0	gas temperature at θ_0 , ($^{\circ}\text{R}$)
t	time, (sec)
V	gas volume, (cu in.)
V_{c1}	clearance volume, (cu in.)
V_D	displacement volume, (cu in.)
θ	crank angle, (radian)
γ	ratio of specific heats
θ_c	effective exhaust-valve closing angle, (radians A.T.C.)
θ'_c	actual exhaust-valve closing angle, (radians A.T.C.)
θ_0	effective exhaust-valve opening angle, (radians A.T.C.)
θ'_0	actual exhaust-valve opening angle, (radians A.T.C.)
$\Delta\theta$	angular increment, (radian)
ϕ	gas-velocity parameter

ANALYSIS

The assumptions necessary to calculate the cylinder pressures during the exhaust process, which have been stated by Komble (reference 1), are:

1. The mixture of gases in the cylinder can be treated as a perfect gas.
2. The process under consideration can be treated as isentropic.

3. The rate of flow through the exhaust valve can be calculated by the ordinary formulas for steady flow through an orifice with the aid of experimental values of the exhaust-valve flow coefficients.

The third assumption requires that the exhaust-manifold pressure be known and because the pressure variations caused by inertia effects in long exhaust pipes cannot be accurately predicted, the exhaust manifold must be of sufficient capacity or the exhaust pipes be sufficiently short that the exhaust pressure P_e remains approximately constant during the exhaust process.

Derivation of differential equation. - The differential equation used to calculate the cylinder pressures during the exhaust process has been developed by several investigators (for example in reference 1), but this equation has been modified to include an original method for treating the exhaust-valve flow coefficient.

The equation of state for a perfect gas is given by

$$\frac{PV}{12} = mRT \quad (1)$$

By taking the differential of equation (1) with respect to the crank angle θ and noting that

$$\frac{dm}{d\theta} = \frac{dm}{dt} \frac{dt}{d\theta}$$

equation (1) becomes

$$\frac{dP}{Pd\theta} = \frac{dT}{Td\theta} + \frac{12RT}{PV} \frac{dm}{dt} \frac{dt}{d\theta} - \frac{dV}{Vd\theta} \quad (2)$$

For the isentropic expansion of the gas within the cylinder, the temperature varies with the pressure as

$$(TP)^{\frac{1-\gamma}{\gamma}} = \text{constant}$$

from which is obtained

$$\frac{dT}{Td\theta} = \left(1 - \frac{1}{\gamma}\right) \frac{dP}{Pd\theta} \quad (3)$$

By combining equations (2) and (3)

$$\frac{dP}{Pd\theta} = \frac{12\gamma RT}{PV} \frac{dm}{dt} \frac{dt}{d\theta} - \frac{\gamma}{V} \frac{dV}{d\theta} \quad (4)$$

The differential $dt/d\theta$ is equal to $60/2\pi N$. Therefore,

$$\frac{dP}{Pd\theta} = \frac{360}{\pi} \frac{\gamma RT}{NV} \frac{dm}{dt} - \frac{\gamma}{V} \frac{dV}{d\theta} \quad (5)$$

The equation relating the flow rate dm/dt to the cylinder pressure P and the exhaust pressure P_e will depend upon the ratio P/P_e . During the first part of the exhaust process, the

values of P/P_e are greater than $\left(\frac{\gamma+1}{2}\right)^{\frac{\gamma}{\gamma-1}}$; therefore, the flow through the valve is sonic and the flow rate is given by

$$\frac{dm}{dt} = - C A_e \sqrt{\frac{2\gamma}{\gamma+1} \left(\frac{P^2}{RT}\right) \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} \quad (6)$$

The flow of gas is out of the cylinder so the flow rate, dm/dt is negative.

When equations (5) and (6) are combined

$$\frac{dP}{Pd\theta} = - \frac{360}{\pi} \frac{\gamma}{NV} C A_e \sqrt{\gamma g RT} \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} - \frac{\gamma}{V} \frac{dV}{d\theta} \quad (7)$$

Because T and P are related by

$$T = T_0 \left(\frac{P_0}{P}\right)^{\frac{1-\gamma}{\gamma}}$$

and the speed of sound corresponding to the temperature in the cylinder when the exhaust valve opens is

$$c_0 = \sqrt{\gamma g RT_0}$$

equation (7) becomes

$$\frac{dP}{Pd\theta} = - \frac{360}{\pi NV} \gamma C A_e c_C \left(\frac{P_0}{P} \right)^{\frac{1-\gamma}{2\gamma}} \sqrt{\left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} - \frac{\gamma}{V} \frac{dV}{d\theta} \quad (8)$$

The cylinder volume V at the crank angle θ is

$$V = V_D (f_1 + f_2) \quad (9)$$

and the change in cylinder volume with crank angle is given by

$$\frac{dV}{d\theta} = V_D f_3 \quad (10)$$

The factors f_2 and f_3 are functions of the crank angle and the ratio of crank throw to connecting-rod length s/l . Values of f_2 and f_3 can be obtained from engineering handbooks.

When equations (9) and (10) are substituted in equation (8)

$$\frac{dP}{Pd\theta} = - \frac{360 C A_e c_C}{\pi N V_D} \left(\frac{\gamma}{f_1 + f_2} \right) \left(\frac{P_0}{P} \right)^{\frac{1-\gamma}{2\gamma}} \sqrt{\left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}} - \frac{\gamma f_3}{f_1 + f_2} \quad (11)$$

Equation (11) is the differential equation that relates cylinder pressure during the first part of the exhaust process to the design characteristics, the engine operating conditions, and the crank angle of the engine.

During the second part of the exhaust process, the values of

P/P_e are less than $\left(\frac{\gamma+1}{2} \right)^{\frac{\gamma}{\gamma-1}}$. Consequently, flow through the valve is subsonic and the flow rate corresponding to equation (6) is

$$\frac{dm}{dt} = - C A_e \sqrt{\frac{2\gamma}{\gamma-1} \frac{P^2}{RT} \left[\left(\frac{P_e}{P} \right)^{\frac{2}{\gamma}} - \left(\frac{P_e}{P} \right)^{\frac{\gamma+1}{\gamma}} \right]} \quad (12)$$

Substitution of this equation into equation (5) and introduction of the same modifications used to derive equation (11) result in

$$\frac{dP}{Pd\theta} = - \frac{360 C A_e c_0}{\pi N V_D} \left(\frac{\gamma}{f_1 + f_2} \right) \left(\frac{P_0}{P} \right)^{\frac{1-\gamma}{2\gamma}} \sqrt{\frac{2}{\gamma-1} \left[\left(\frac{P_0}{P} \right)^{\frac{2}{\gamma}} - \left(\frac{P_e}{P} \right)^{\frac{\gamma+1}{\gamma}} \right]} - \frac{\gamma f_3}{f_1 + f_2} \quad (13)$$

Equation (13) is the differential equation that relates cylinder pressure during the second part of the exhaust process to the design characteristics, the engine operating conditions, and the crank angle of the engine.

Exhaust-valve flow coefficient C_e . - The exhaust-valve flow coefficient for any valve lift is determined under steady-flow conditions by measuring the rate of air flow through the exhaust valve and port with a fixed pressure drop across the valve. The exhaust-valve flow coefficient is then defined as the measured rate of air flow divided by the calculated rate of air flow based upon the nominal valve area. The nominal valve area is the same for all valve lifts and is equal to $\pi d^2/4$. The value of d selected is immaterial inasmuch as the product CA_e used in the calculations is independent of the valve diameter so long as the values of C and A_e are based on the same diameter.

A complete description of valve-flow test methods and apparatus is given in reference 2. The conventional method for plotting valve-flow coefficient data produces a curve of valve-flow coefficient C against the ratio of valve lift to valve diameter L/d . Such a curve is presented in figure 1 for engine A, which is a typical high-speed aircraft poppet-valve engine used as an example in this report. For every valve lift therefore a valve-flow coefficient exists; a curve of valve-flow coefficient against engine crank angle can be plotted if the variation in valve lift with crank angle is known. Such a curve has been plotted in figure 2 for engine A.

The valve-flow coefficients obtained under steady-flow conditions can be used to calculate the cylinder pressure during the exhaust process (intermittent-flow conditions) because (1) the steady- and intermittent-flow coefficients of poppet valves are not appreciably different (reference 3) and (2) the flow coefficient of exhaust valves does not vary appreciably with the pressure drop across the valve (reference 4).

In order to express the variation in exhaust-valve flow coefficient with engine crank angle a sine-curve approximation of the exhaust-valve flow coefficient can be closely fitted to the actual flow-coefficient curve of nearly all high-speed aircraft engines. For those few engines with a variation in exhaust-valve flow coefficient that cannot be approximated by a sine curve, the quantitative results of this analysis are inapplicable although most of the trends and conclusions probably can be applied. In order to fit the sine curve to the actual flow-coefficient curve, it is assumed that:

- (1) the value of the flow coefficient at the maximum valve lift is the same in both cases
- (2) the total area under the curve of flow coefficient plotted against crank angle is the same in both cases

From assumption (1) it follows that

$$C = C_{\max} \sin \left[\pi \left(\frac{\theta - \theta_0}{\theta_c - \theta_0} \right) \right] \quad (14)$$

The values of θ_0 and θ_c depend upon assumption (2) and upon the actual opening and closing angles θ'_0 and θ'_c . The values of θ_0 and θ_c for a specific engine are found as follows:

The area under the curve of the actual flow coefficient plotted against crank angle is

$$\text{Area} = C'_a (\theta'_c - \theta'_0) \quad (15)$$

where C'_a is the average flow coefficient (reference 5). From equation (14) the area under the approximate flow-coefficient curve is

$$\begin{aligned} \text{Area} &= \int_{\theta_0}^{\theta_c} C_{\max} \sin \left[\pi \left(\frac{\theta - \theta_0}{\theta_c - \theta_0} \right) \right] d\theta \\ &= \frac{2 C_{\max}}{\pi} (\theta_c - \theta_0) \end{aligned} \quad (16)$$

By combination of equations (15) and (16)

$$C_a' (\theta_c' - \theta_0') = \frac{2 C_{\max}}{\pi} (\theta_c - \theta_0)$$

or

$$\theta_c - \theta_0 = \frac{\pi}{2} \frac{C_a'}{C_{\max}} (\theta_c' - \theta_0') \quad (17)$$

For a symmetrical cam

$$\theta_0 = \theta_0' + \frac{(\theta_c' - \theta_0') - (\theta_c - \theta_0)}{2} \quad (18)$$

and

$$\theta_c = \theta_c' - \frac{(\theta_c' - \theta_0') - (\theta_c - \theta_0)}{2} \quad (19)$$

By substitution of equation (17) into equations (18) and (19)

$$\theta_0 = \theta_0' + \frac{(\theta_c' - \theta_0')}{2} \left(1 - \frac{\pi}{2} \frac{C_a'}{C_{\max}} \right) \quad (20)$$

and

$$\theta_c = \theta_c' - \frac{(\theta_c' - \theta_0')}{2} \left(1 - \frac{\pi}{2} \frac{C_a'}{C_{\max}} \right) \quad (21)$$

In order to apply the results of this analysis to a specific engine, knowledge of the valve timings θ_0' and θ_c' and the coefficients C_a' and C_{\max} is necessary. A comparison of the actual flow-coefficient curve with the approximate flow-coefficient curve given by equation (14) is shown in figure 2 for a typical aircraft engine (engine A).

Final equations. - By substitution of equation (14) into equations (11) and (13), by division of the pressures in both equations by the exhaust pressure P_0 , and because the quantity FV_D is equal to $6 SA_p$ the following equations are obtained:

For the first part of the exhaust process

$$\frac{d}{d\theta} \left(\frac{P}{P_e} \right) = - \frac{\gamma}{f_1 + f_2} \left(\frac{P}{P_e} \right)$$

$$\left\{ \frac{1}{2\phi} \left[\left(\frac{P_0}{P_e} \right) \left(\frac{P_e}{P} \right) \right]^{\frac{1-\gamma}{2\gamma}} \sin \left[\pi \left(\frac{\theta - \theta_0}{\theta_c - \theta_0} \right) \right] \sqrt{\left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} + f_3 \right\} \quad (22)$$

For the second part of the exhaust process

$$\frac{d}{d\theta} \left(\frac{P}{P_e} \right) = - \frac{\gamma}{f_1 + f_2} \left(\frac{P}{P_e} \right)$$

$$\left\{ \frac{1}{2\phi} \left[\left(\frac{P_0}{P_e} \right) \left(\frac{P_e}{P} \right) \right]^{\frac{1-\gamma}{2\gamma}} \sin \left[\pi \left(\frac{\theta - \theta_0}{\theta_c - \theta_0} \right) \right] \sqrt{\frac{2}{\gamma - 1} \left[\left(\frac{P_e}{P} \right)^{\frac{2}{\gamma}} - \left(\frac{P_e}{P} \right)^{\frac{\gamma+1}{\gamma}} \right]} + f_3 \right\} \quad (23)$$

where

$$\phi = \frac{\pi SA_p}{120 c_0 C_{max} A_e}$$

and has the characteristics of a Mach number.

Gas-velocity parameter. - The gas-velocity parameter ϕ is important because it relates engine speed to exhaust-valve area. For example, from equations (22) and (23) it is evident that for the same initial conditions of exhaust-valve opening engine speed will have no effect upon the gas-pressure ratio P/P_e during the exhaust stroke provided that the exhaust-valve area and the flow coefficient are so changed that ϕ remains constant. The significance of ϕ becomes apparent if $2 C_{max}/\pi$ is replaced by the average value of the flow coefficient for the sine-curve approximation C_a . If this substitution is made, ϕ becomes equal to

$$\frac{SA_p}{60 c_0 C_a A_e}$$

The effect of this parameter on volumetric efficiency when applied to the intake system has been demonstrated in reference 5. The parameter is probably equally important when applied to the exhaust system.

Representative values of the gas-velocity parameter ϕ are given in the following table for several military aircraft engines at take-off speeds. The values are based upon a value of c_0 equal to 2500 feet per second.

Engine	Average piston speed (S)	Ratio of piston area to exhaust-valve area (A_p/A_e)	Maximum exhaust-valve flow coefficient (C_{max})	Gas-velocity parameter (ϕ)
A	3000	5.73	0.608	0.297
B	2860	4.04	.440	.275
C	2800	4.90	.493	.292

METHOD OF CALCULATION

Inasmuch as differential equations (22) and (23) relating gas-pressure ratio to design characteristics, operating conditions, and crank angle cannot be integrated, it is necessary to resort to an approximate point-by-point solution. In order to demonstrate this method of calculating the gas-pressure ratios during the exhaust process, an example has been calculated for the following assumed conditions:

Initial gas-pressure ratio at θ_0 , P_0/P_e	9.33
Gas-velocity parameter, ϕ	0.325
Compression ratio, r	7.5
Effective exhaust-valve opening, θ_0 , degrees A.T.C.	110
Effective exhaust-valve closing, θ_c , degrees A.T.C.	20
Ratio of specific heats, γ	1.31
Crank throw to connecting-rod length ratio, s/l	0.250

The variation of gas-pressure ratio with crank angle during the first part of the exhaust process is calculated by equation (22) for sonic flow through the exhaust valve. The change in gas-pressure ratio with crank angle at the effective exhaust-valve-opening angle

is calculated by equation (22). Multiplication of the slope $\frac{d}{d\theta} \left(\frac{P}{P_e} \right)$ by a small angular increment gives the change in cylinder pressure that occurs during that angular increment. The resulting pressure at the new angle is then known and the slope at the new angle is calculated by use of equation (22). The process is repeated until a crank angle is reached at which the cylinder pressure is sufficiently low that the flow through the exhaust valve becomes subsonic and equation (23) must be used to calculate the slope. The value of the gas-pressure ratio at which the flow becomes subsonic is

$$\frac{P}{P_e} = \left(\frac{\gamma + 1}{2} \right)^{\frac{\gamma}{\gamma - 1}}$$

For the value of the ratio of specific heats γ assumed in this example, the value of P/P_e is 1.84. The calculations are presented in table I for the first part of the exhaust process.

When the gas-pressure ratio becomes less than 1.84, the rest of the exhaust process must be calculated by equation (23) for subsonic flow through the exhaust valve. Calculations given for this part of the exhaust process are listed in table II. The curve of gas-pressure ratio against crank angle from tables I and II is plotted in figure 3.

RESULTS AND DISCUSSION

An inspection of equations (22) and (23) and their derivations show that the shape of the curve of gas-pressure ratio (cylinder pressure) plotted against crank angle during the exhaust process is dependent upon the following factors:

- Gas-velocity parameter, ϕ
- Gas-pressure ratio, P_0/P_e
- Effective exhaust-valve closing angle, θ_c
- Effective exhaust-valve opening angle, θ_0
- Compression ratio, r
- Crank throw to connecting-rod length ratio, s/l

The ratio of specific heats γ was considered to be invariable and a value of 1.31 was used, which is approximately the average value of γ for exhaust gases at the temperatures existing during the exhaust process (reference 6).

Each of these factors was investigated by varying its value over a suitable range while the remaining factors were held constant at the following values:

Gas-velocity parameter, ϕ	0.225, 0.275, 0.325, and 0.375
Gas-pressure ratio, P_0/P_e	9.33
Effective exhaust-valve closing angle, θ_c , degrees A.T.C.	5
Effective exhaust-valve opening angle, θ_0 , degrees A.T.C.	110
Compression ratio, r	7.5
Crank throw to connecting-rod length ratio, s/l	0.250

Because of its importance, the factor ϕ has been investigated over a range of values in all cases.

Inasmuch as the gas-pressure ratio in the cylinder near top center during the exhaust stroke is indicative of the excellence of the exhaust process, the effect of these factors on the exhaust process is indicated by plotting the gas-pressure ratio 340° A.T.C. P_{340}/P_e against the factor being investigated. The angle 340° A.T.C. was selected rather than top center because for nearly all aircraft engines the intake valve is not open or has not been open sufficiently long at this point to affect the calculated value of the cylinder pressure.

Initial gas-pressure ratio, P_0/P_e . - The effect of the initial gas-pressure ratio P_0/P_e on P_{340}/P_e is given in figure 4. The range of initial gas-pressure ratios investigated is considered to be sufficient to cover conditions encountered by compounded reciprocating engines operating with high exhaust pressures (low values of P_0/P_e) and by highly supercharged reciprocating engines operating at high altitudes (high values of P_0/P_e). From an inspection of various indicator cards (reference 7), it was found that the standard value of P_0/P_e equal to 9.33 at an engine crank angle of 110° A.T.C. corresponds to a value of P_m/P_e approximately equal to 1.7. The value of P_0/P_e for a given effective exhaust-valve opening angle should vary nearly in proportion to the value of P_m/P_e because the magnitude of P_0 varies in proportion to the air consumption which, in turn, is nearly proportional to P_m .

From figure 4 it appears that for low gas-velocity parameters (ϕ equals 0.225 and 0.275), P_0/P_e has very little effect on

P_{340}/P_e for values of P_0/P_e up to 15, which for an engine crank angle of 110° A.T.C. corresponds to a value of P_m/P_e equal to approximately 2.75. Even for values of ϕ corresponding to take-off speeds (ϕ equals approximately 0.300, table I) P_0/P_e has an unimportant effect upon P_{340}/P_e for values of P_0/P_e up to 10 or 12, which for an engine crank angle of 110° A.T.C. corresponds to values of P_m/P_e equal to approximately 1.8 to 2.2. Therefore, for most of the practical engine operating conditions the intake manifold pressure P_m and the intake-valve area A_e , which determine the value of P_0 , have little effect upon the value of P_{340}/P_e . Consequently, there is no correct ratio of intake-to-exhaust-valve area nor is the exhaust-valve size dependent upon the ratio of intake-to-exhaust pressure. However, figure 4 also indicates that for gas-velocity parameters somewhat greater than those currently in practice (values of ϕ equal to 0.325 or higher) the initial gas-pressure ratio P_0/P_e can have a very significant effect on the value of P_{340}/P_e and therefore the ratio of intake-to-exhaust-valve area and the ratio of intake-to-exhaust pressure may be significant considerations under these conditions.

Effective exhaust-valve closing angle, θ_c . - The influence of the effective exhaust-valve closing angle θ_c on the gas-pressure ratio P_{340}/P_e is shown in figure 4, which is plotted on reciprocal-coordinate paper. A comparison of these curves shows that for all gas-velocity parameters (ϕ equals 0.225, 0.275, 0.325, and 0.375) the gas-pressure ratio P_{340}/P_e decreases with increasing values of θ_c . The significance of the effective exhaust-valve closing angle on the back pressure in the cylinder at the intake-valve opening is exhibited by the rapid decrease in P_{340}/P_e with increasing values of θ_c , especially at the high speeds (ϕ equal to 0.325 and 0.375). The significant conclusion to be drawn from these curves is that for high-speed operation of a reciprocating internal-combustion engine, the exhaust valve should be closed as late as possible to promote efficient cylinder charging. Not only is the gas-pressure ratio P_{340}/P_e lower under these conditions but the time available for scavenging the clearance volume is so increased that it appears doubly advantageous to close the exhaust valve late.

Effective exhaust-valve opening angle, θ_o . - The influence of the effective exhaust-valve opening angle θ_o on the gas-pressure ratio P_{340}/P_e is given in figure 5. For low values of

the gas-velocity parameter (ϕ equals 0.225 and 0.275), late opening of the exhaust valve results in decreasing values of P_{340}/P_e . This characteristic exists because the exhaust valve is further open at a crank angle θ of 340° A.T.C. when the effective exhaust-valve opening angle occurs near the bottom center. (Note that the effective exhaust-valve-closing angle is fixed.) At high engine speeds, however, sufficient time is not available for the exhaust gases to flow out of the cylinder; therefore, although the exhaust valve is further open at 340° A.T.C., the cylinder pressure is higher than it would have been had the exhaust valve opened near bottom center. This characteristic is shown in figure 5 by the curve for ϕ equal to 0.325. For this particular value of ϕ , P_{340}/P_e at first falls off with increasing values of θ_0 because the exhaust valve is further open at 340° ; when θ_0 occurs too close to bottom center, however, the time available for the exhaust gases to flow out of the cylinder is too short and the value of P_{340}/P_e increases.

For very high values of the gas-velocity parameter (ϕ equals 0.375) the value of P_{340}/P_e increases with increasing values of θ_0 for all values of θ_0 investigated; therefore, for these gas-velocity parameters it is advisable to open the exhaust valve as early as possible to promote efficient cylinder charging. However, for values of ϕ less than 0.350, changes in the effective exhaust-valve opening angle have little effect upon the value of P_{340}/P_e and therefore on cylinder charging. The value of θ_0 selected will be determined by other factors such as blowdown and pumping losses, which can also be calculated by the equations for gas pressure given in this report.

Compression ratio, r . - The effect of change in compression ratio r on the gas-pressure ratio P_{340}/P_e is shown in figure 6. For low gas-velocity parameters (ϕ equals 0.225 and 0.275), r has no effect upon P_{340}/P_e , but for gas-velocity parameters slightly above current take-off speeds (ϕ equals 0.325), compression ratio begins to have some effect, the value of P_{340}/P_e decreasing with increasing compression ratio. For very high gas-velocity parameters (ϕ equals 0.375), the value of P_{340}/P_e decreases considerably with increasing compression ratio and it is therefore advantageous to use a high compression ratio. For the gas-velocity parameters currently used in military aircraft engines, however, the effect of the compression ratio on the cylinder pressures is so insignificant that the compression ratio need not be considered in determination of the correct size and timing of the exhaust valve.

Ratio of crank throw to connecting-rod length, s/l . - The value of s/l used in the calculations is 0.250. Values of s/l used in several military aircraft engines are given in the following table:

Engine	Crank throw s (in.)	Connecting-rod length l (in.)	Ratio of crank throw to connecting-rod length s/l
A	3.000	10.000	0.300
B	3.438	13.750	.250
C	3.000	12.312	.244

The s/l ratios do not vary greatly among engines and as the ratio has little effect on piston motion, no cylinder-pressure curves have been calculated for different values of s/l .

Relative significance of factors affecting exhaust-valve size. - As a result of the foregoing analysis, the factors that affect the process can be classified as significant or insignificant. These factors have been grouped in the following list in the order of significance:

Significant factors that must be considered

1. Gas-velocity parameter, ϕ
2. Gas-pressure ratio, P_0/P_e
3. Effective exhaust-valve closing angle, θ_c

Insignificant factors for values of ϕ less than approximately 0.350

1. Effective exhaust-valve opening angle, θ_o
2. Compression ratio, r
3. Crank throw to connecting-rod length ratio, s/l

Recommended values of gas-velocity parameter, ϕ . - The gas-velocity parameter ϕ interrelates the exhaust-valve size with the piston speed, piston area, and the exhaust-valve flow coefficient at maximum valve lift. Thus, if a maximum value of ϕ can be recommended, a maximum value of engine speed for a given valve size or the minimum size of exhaust valve required for a given engine speed is known.

According to the values of P_{340}/P_e obtained from figures 4 to 6, the cylinder-charging process would be adversely affected by values of ϕ greater than 0.325 because of the high values of P_{340}/P_e that exist for these gas-velocity parameters. The recommendation is therefore made that the maximum value of ϕ be no higher than 0.300 to 0.325.

ACCURACY OF RESULTS

The accuracy with which the calculated results of this analysis can be used may be affected by one or all of the following factors:

1. Effects of exhaust pipes
2. Selected value of ratio of specific heats, γ
3. Angular increment used in the point-by-point solution of the differential equation
4. Sine-curve approximation of the exhaust-valve flow coefficient, C

The assumption previously stated, which requires that the exhaust manifold be of sufficient capacity or the exhaust pipes be sufficiently short in order that the exhaust pressure P_e remains approximately constant during the exhaust process, must be satisfied because inertia effects in long exhaust pipes can completely change the magnitude of the cylinder pressures during the entire exhaust process.

The effect of the selected value of ratio of specific heats γ on the gas-pressure ratio P_{340}/P_e is shown in figure 7 for two values of the gas-velocity parameter ϕ . For low values of gas-velocity parameter ϕ the value of ratio of specific heats γ has no significant effect on gas-pressure ratio P_{340}/P_e but for higher values the effect becomes significant. Because the gas temperatures and composition are sufficiently well known to enable a fairly accurate selection of ratio of specific heats γ (1.31 has been selected based on material presented in reference 6), no significant error should result from a small error in the selected value of γ .

The effect of angular increment $\Delta\theta$ on the calculated values of gas-pressure ratio P_{340}/P_e is shown in figure 8 for the complete

range of gas-velocity parameter ϕ . For low values of the gas-velocity parameter ϕ , which result in low values of gas-pressure ratio P_{340}/P_e , the error introduced by calculating the pressure curve with 10° intervals is of no consequence; however, large values of gas-velocity parameter ϕ (and high values of initial gas-pressure ratio P_0/P_e) generally result in relatively large rates of change in cylinder pressure with change in crank angle near the end of the exhaust stroke and therefore errors caused by the point-by-point solution of the differential equation become insignificant under these conditions. As all calculations for this report were made using 10° intervals, calculated values of gas-pressure ratio P_{340}/P_e that are large can be expected to be somewhat lower than would be obtained by an exact solution of the differential equation.

In order to show the effect of the sine-curve approximation on the curve of gas-pressure ratio plotted against crank angle, calculations using the actual and the approximate values of the flow coefficients for engine A are plotted in figure 9. No significant difference in the curves exists; however, earlier exhaust-valve closing angles than those used on engine A result in relatively large differences between the actual and the approximate values of the flow coefficient C at the crank angle corresponding to gas-pressure ratio P_{340}/P_e ; therefore, errors caused by the valve-flow-coefficient approximation can be significant if the effective exhaust-valve closing angle θ_c occurs near top center. For the same reason, the results of this analysis would be inaccurately applied to those few engines having a variation in exhaust-valve flow coefficient that cannot be closely approximated by the sine curve.

APPLICATION OF RESULTS

Although the principal purpose of this analysis was to determine the relative significance of the various factors that affect the exhaust process, the results can also be used to estimate the cylinder pressure at 20° B.T.C. The cylinder pressure at this crank angle is of interest because high values indicate high cylinder back pressures at intake-valve opening and may therefore explain low values of volumetric efficiency.

As an example, the cylinder pressure in engine A at 20° B.T.C. during the exhaust process will be estimated from the plots for an initial gas-pressure ratio P_0/P_e equal to 9.33 and for an engine speed of 3200 rpm. Under these conditions the value of gas-velocity

parameter ϕ is 0.317. The exhaust-valve closing angle θ_c^0 is 26° A.T.C. and the exhaust-valve opening angle θ_o^0 is 104° A.T.C.; inasmuch as the values of actual exhaust-valve flow coefficient C_a and the maximum exhaust-valve flow coefficient C_{max} are 0.373 and 0.608, respectively, the value of effective exhaust-valve closing angle θ_c calculated by equation (21) is 21° . From figure 4(b) the calculated value of P_{340}/P_e is found to be 1.22.

Results of this analysis can also be used to determine the effect on cylinder charging of changing the exhaust-valve timing by shifting the exhaust-valve cam. In this case both the effective opening angle and the effective closing angle are changed and all other factors remain the same. The analysis shows that the change in effective exhaust-valve opening angle is insignificant (from the standpoint of cylinder charging) and therefore only the change in effective exhaust-valve closing angle need be considered and the magnitude of its effect may be obtained from figure 4.

CONCLUSIONS

If the variation in exhaust-valve flow coefficient with engine crank angle can be approximated by a sine curve and the effects of exhaust pipes and manifolds can be neglected, the following conclusions can be made as a result of an analysis of the exhaust process of a typical high-speed poppet-valve engine:

1. The factors that have a significant effect upon the exhaust process are gas-velocity parameter, initial gas-pressure ratio, and effective exhaust-valve closing angle.
2. The factors that have an insignificant effect upon the exhaust process (except perhaps at extremely high speeds) are effective exhaust-valve opening angle, compression ratio, and ratio of crank throw to connecting-rod length.
3. The recommended maximum value for the gas-velocity parameter is 0.325.
4. For large values of the gas-velocity parameter (above 0.275) the best method of reducing the value of the gas-pressure ratio at 340° A.T.C. is to close the exhaust valve late.
5. For most practical engine operating conditions, the intake-valve area has an insignificant effect upon the exhaust process, therefore there is no correct ratio of intake-to exhaust-valve area.

6. For most practical engine operating conditions the intake-manifold pressure does not have a significant effect upon the exhaust process.

Aircraft Engine Research Laboratory,
National Advisory Committee for Aeronautics,
Cleveland, Ohio, October 31, 1946.

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TABLE I - CALCULATED GAS-PRESSURE RATIO DURING FIRST PART OF EXHAUST PROCESS

1	2	3	4	5	6	7	8
θ (deg A.T.C.)	P/P_θ	$\left[\left(\frac{P_0}{P_\theta}\right)\left(\frac{P_\theta}{P}\right)\right]^{\frac{1-\gamma}{2\gamma}}$	$\frac{\sin\left[\pi\left(\frac{\theta-\theta_0}{\theta_c-\theta_0}\right)\right]}{f_1+f_2}$	$(K_1)_{(a)}(3)(4)$	$-\frac{\gamma f_3}{f_1+f_2}$	(6) - (5)	$\Delta \frac{P}{P_\theta} =$ (7)(2)($\Delta\theta$)
110	9.33	1.000	0.000	0.000	-0.636	-0.636	-0.20
112	9.13	.996	.026	.030	-.612	-.642	-.31
115	8.82	.992	.063	.073	-.577	-.650	-.50
120	8.32	.988	.122	.142	-.520	-.662	-.96
130	7.36	.972	.228	.260	-.414	-.674	-.87
140	6.49	.956	.321	.361	-.319	-.630	-.77
150	5.72	.943	.407	.451	-.233	-.684	-.68
160	5.04	.930	.485	.530	-.150	-.680	-.61
170	4.43	.915	.559	.601	-.075	-.676	-.53
180	3.90	.902	.630	.668	.000	-.668	-.45
190	3.45	.889	.698	.729	.075	-.654	-.40
200	3.05	.877	.765	.788	.150	-.638	-.34
210	2.71	.864	.832	.845	.233	-.612	-.28
220	2.43	.853	.901	.903	.319	-.584	-.25
230	2.18	.843	.973	.964	.414	-.550	-.21
240	1.97	.833	1.050	1.028	.520	-.508	-.17
250	1.80						

$$a_{K_1} = \frac{\gamma}{2\phi} \sqrt{\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}} = 1.175.$$

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TABLE II - CALCULATED GAS-PRESSURE RATIO DURING SECOND PART OF EXHAUST PROCESS

1	2	3	4	5	6	7	8
θ (deg A.T.C.)	P/P_e	ψ (a)	$\frac{\sin \left[\pi \left(\frac{\theta - \theta_0}{\theta_c - \theta_0} \right) \right]}{f_1 + f_2}$	$(K_2)(3)(4)$ (b)	$-\frac{\gamma f_3}{f_1 + f_2}$	$(6) - (5)$	$\Delta \frac{P}{P_e} =$ $(7)(2)(\Delta \theta)$
250	1.800	0.1894	1.133	1.098	0.636	-0.462	-0.145
260	1.655	.1860	1.228	1.170	.770	-.400	-.116
270	1.539	.1810	1.335	1.237	.912	-.325	-.082
280	1.457	.1755	1.461	1.309	1.072	-.237	-.061
290	1.396	.1706	1.609	1.405	1.241	-.164	-.038
300	1.358	.1666	1.781	1.519	1.419	-.100	-.024
310	1.334	.1638	1.970	1.652	1.580	-.072	-.018
320	1.316	.1614	2.171	1.794	1.692	-.102	-.023
330	1.293	.1585	2.328	1.886	1.686	-.202	-.045
340	1.248						

$$a. \psi = \left[\left(\frac{P_0}{P_e} \right) \left(\frac{P_e}{P} \right)^{\frac{1-\gamma}{2\gamma}} \right] \sqrt{\left(\frac{P_e}{P} \right)^{\frac{2}{\gamma}} - \left(\frac{P_e}{P} \right)^{\frac{\gamma+1}{\gamma}}}$$

$$b. K_2 = \frac{\gamma}{2\phi} \sqrt{\frac{2}{\gamma-1}} = 5.120$$

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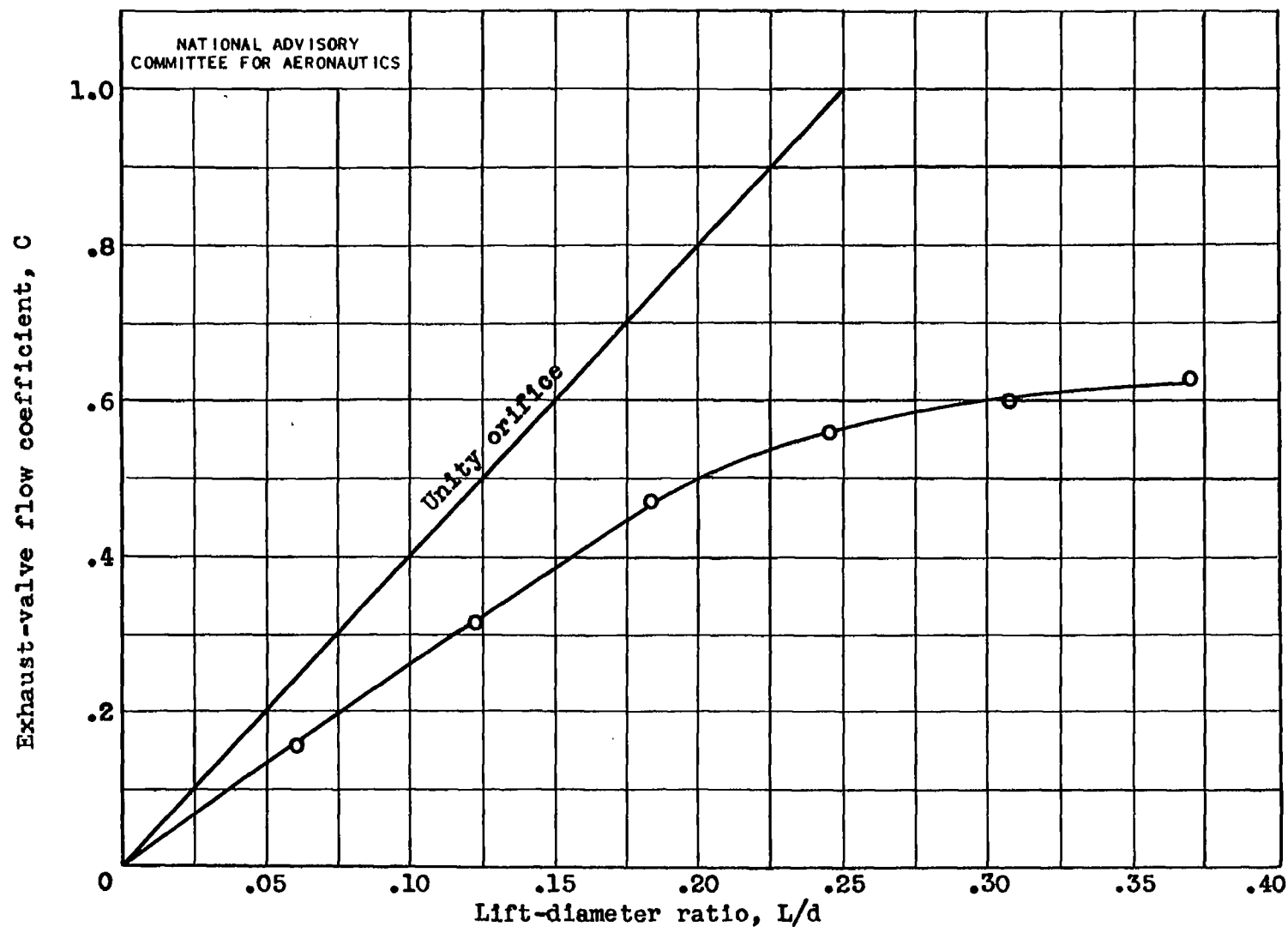


Figure 1. - Change in exhaust-valve flow coefficient with lift-diameter ratio. Engine A; nominal valve diameter, 1.625 inches.

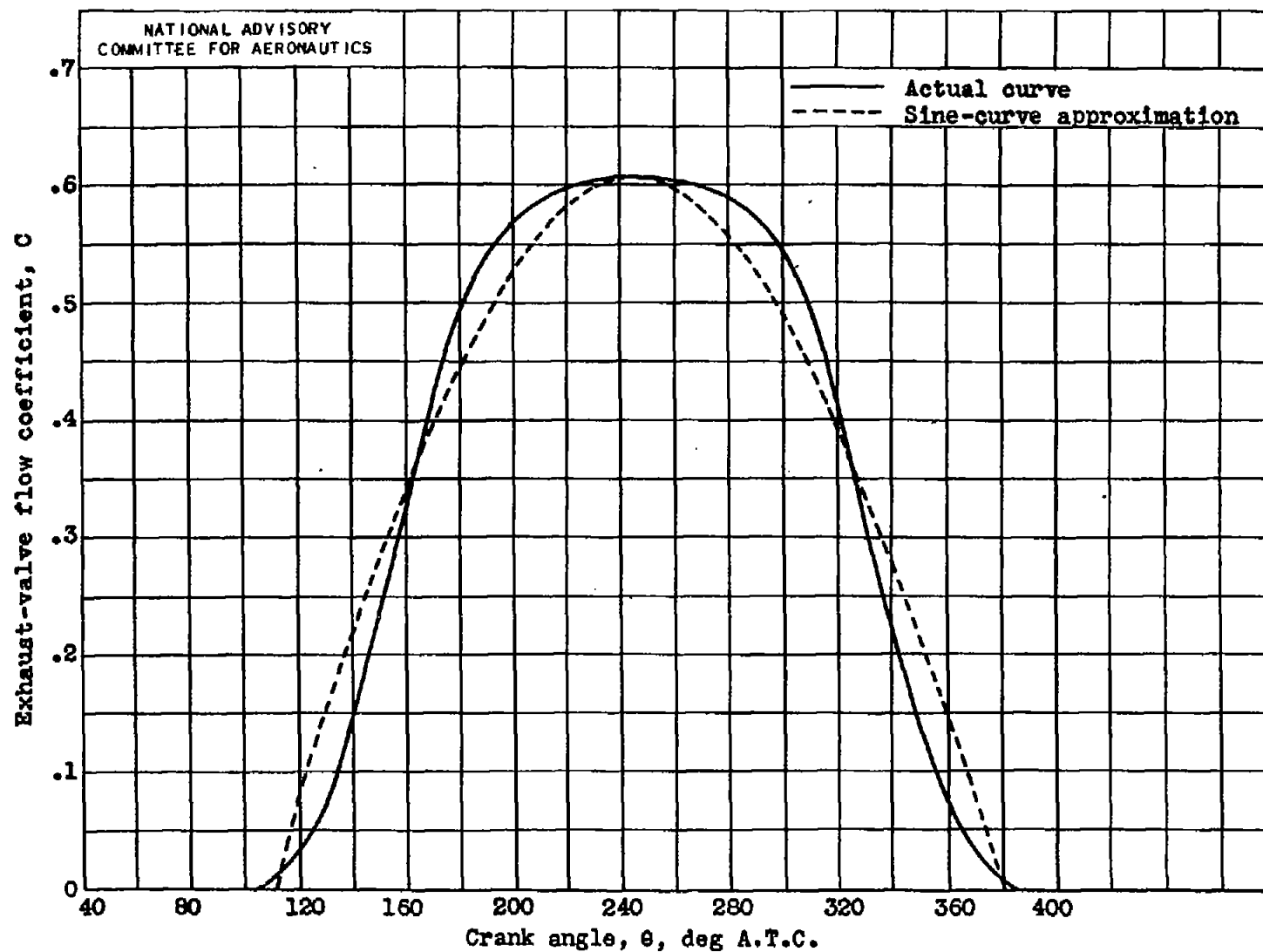


Figure 2. - Comparison of actual exhaust-valve flow-coefficient curve with sine-curve approximation. Engine A.

Fig. 2

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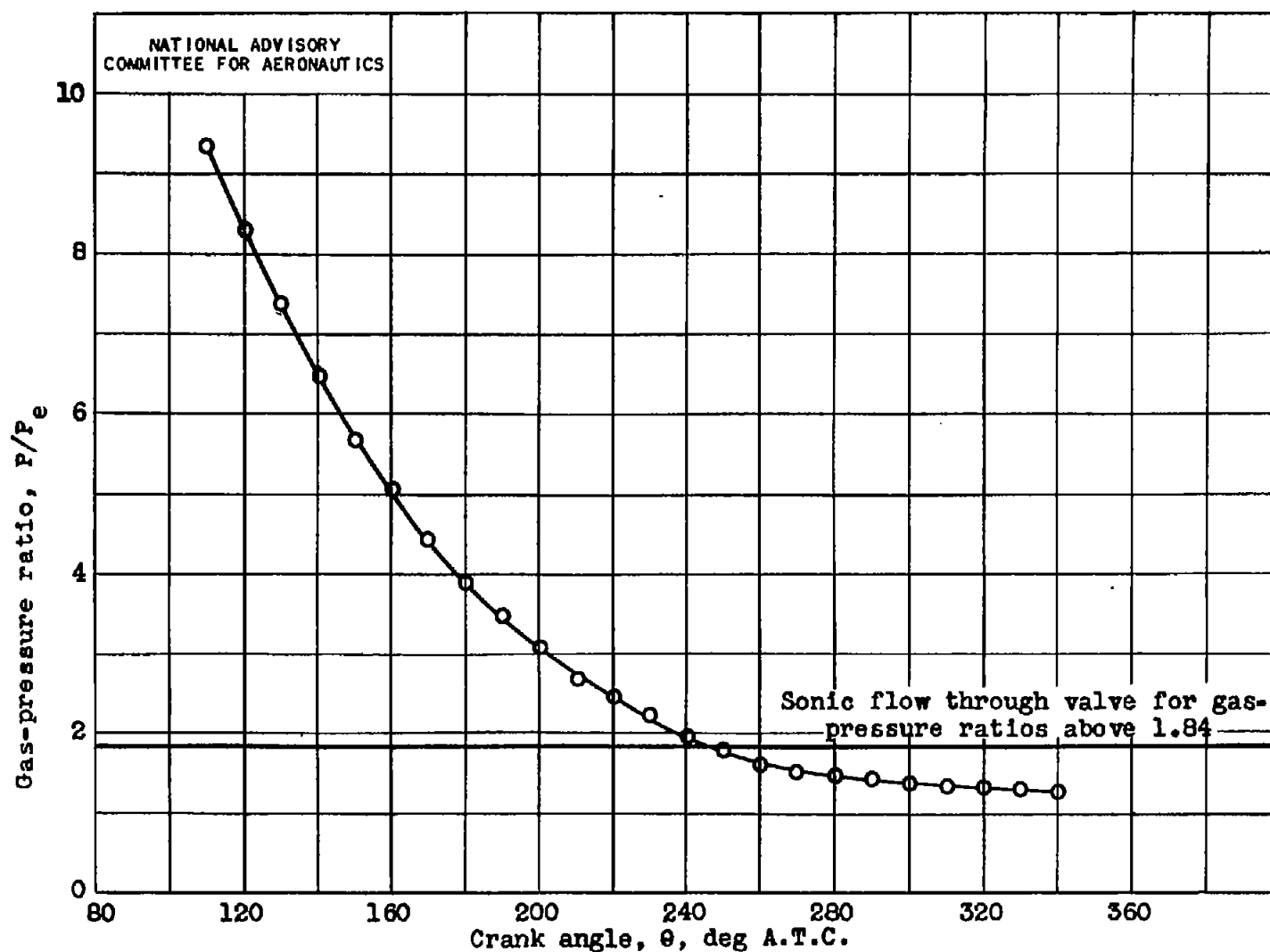


Figure 3. - Typical curve of gas-pressure ratio variation with crank angle. Effective opening angle, 110° A.T.C.; effective closing angle, 20° A.T.C.; initial gas-pressure ratio, 9.33; compression ratio, 7.5; gas-velocity parameter, 0.325.

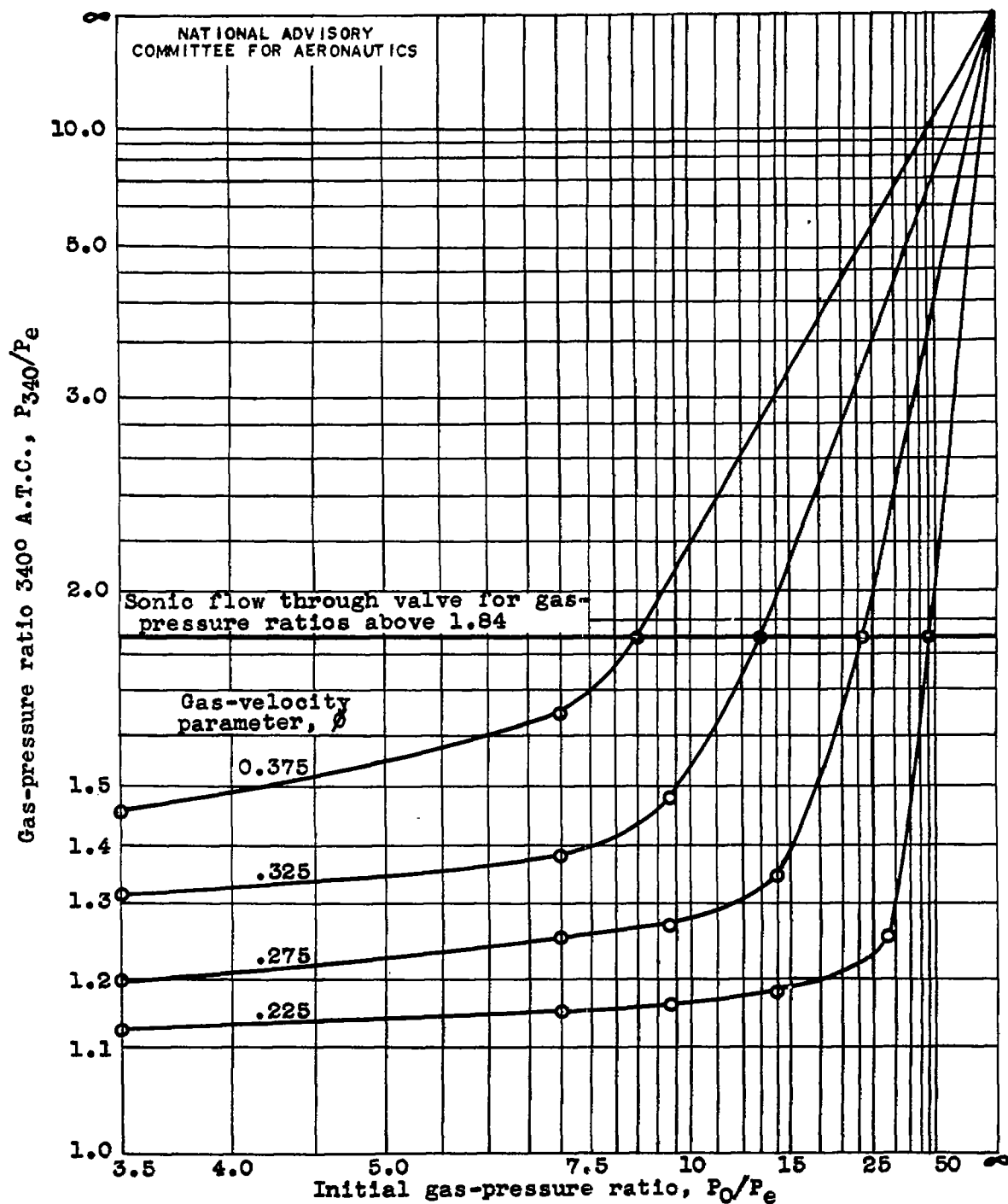
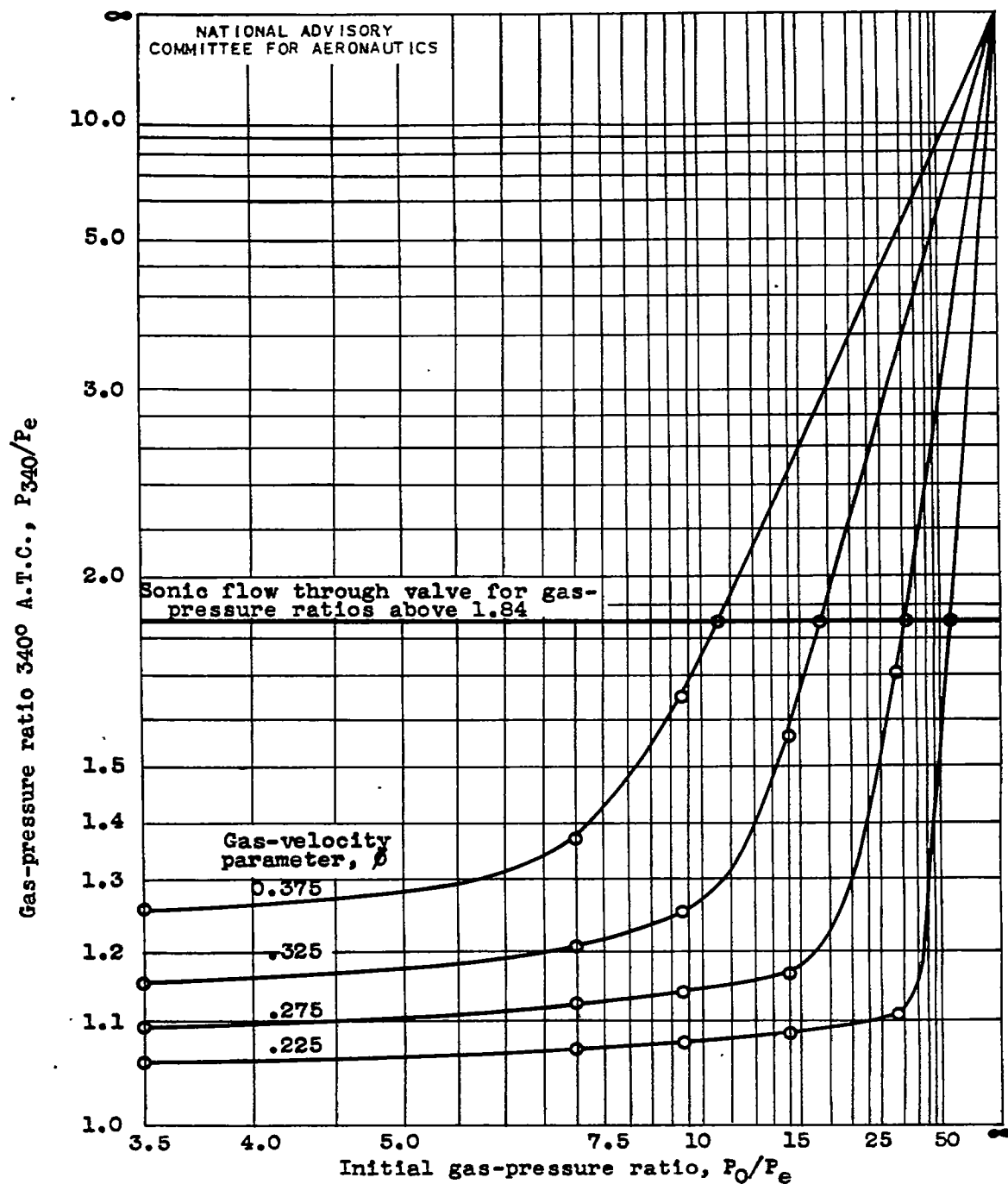
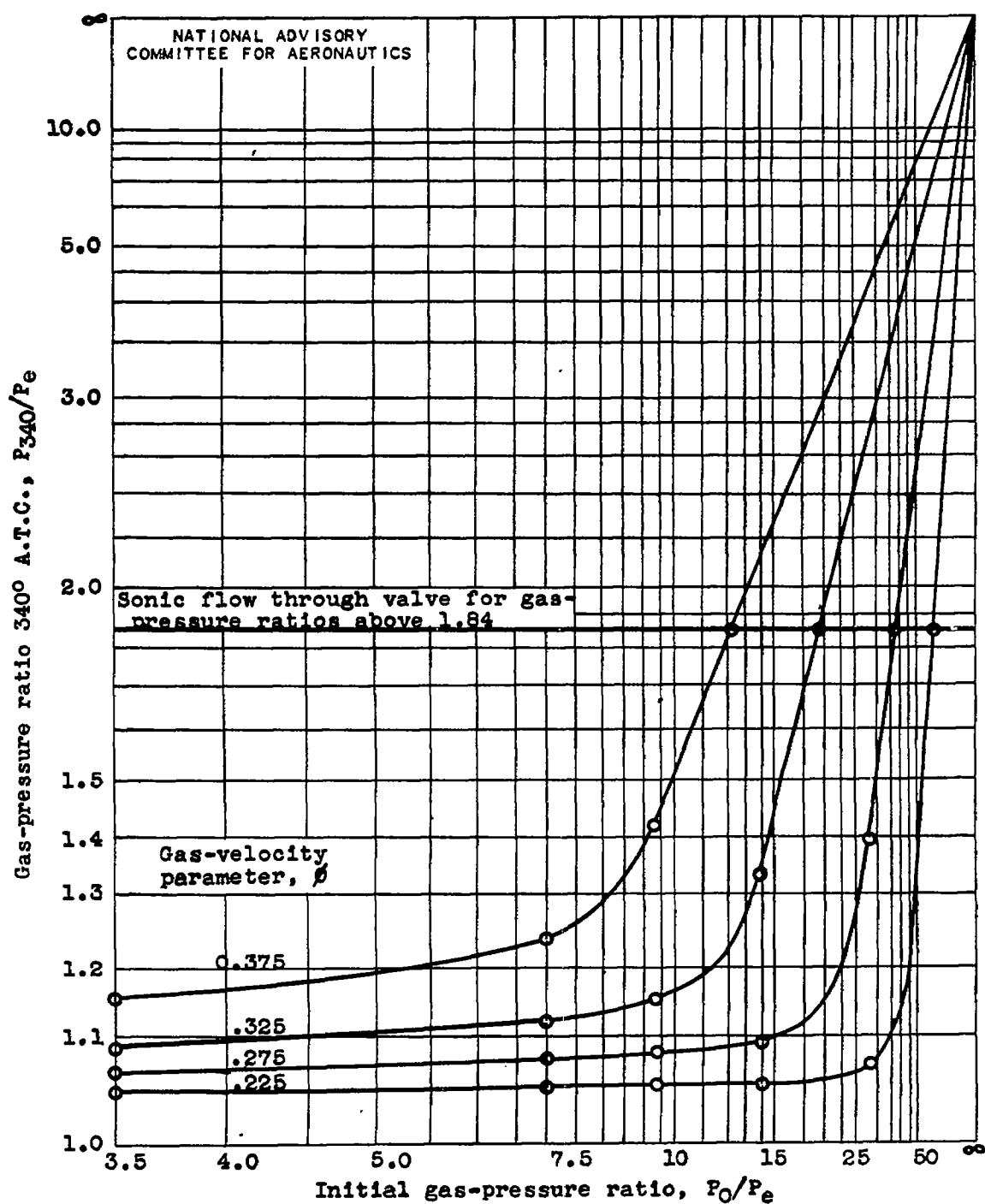
(a) Effective closing angle, θ_0 , 5° A.T.C.

Figure 4. - Variation in gas-pressure ratio 340° A.T.C. with initial gas-pressure ratio for various values of gas-velocity parameter. Effective opening angle, 110° A.T.C.; compression ratio, 7.5



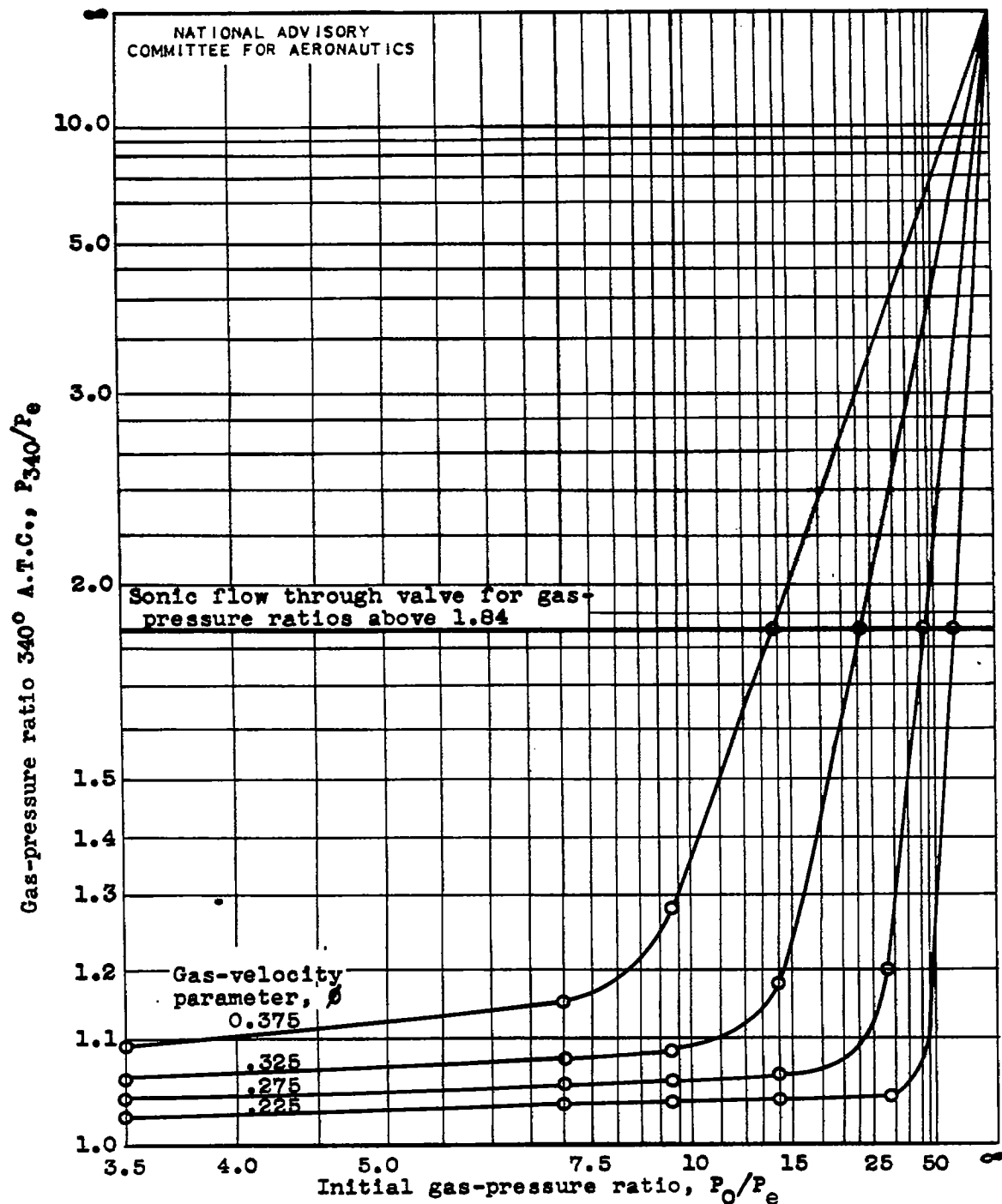
(b) Effective closing angle, θ_c , 20° A.T.C.

Figure 4. - Continued. Variation in gas-pressure ratio 340° A.T.C. with initial gas-pressure ratio for various values of gas-velocity parameter. Effective opening angle, 110° A.T.C.; compression ratio, 7.5.



(c) Effective closing angle, θ_c , 35° A.T.C.

Figure 4. - Continued. Variation in gas-pressure ratio 340° A.T.C. with initial gas-pressure ratio for various values of gas-velocity parameter. Effective opening angle, 110° A.T.C.; compression ratio, 7.5.



(d) Effective closing angle, θ_c , 50° A.T.C.

Figure 4. - Concluded. Variation in gas-pressure ratio 340° A.T.C. with initial gas-pressure ratio for various values of gas-velocity parameter. Effective opening angle, 110° A.T.C.; compression ratio, 7.5.

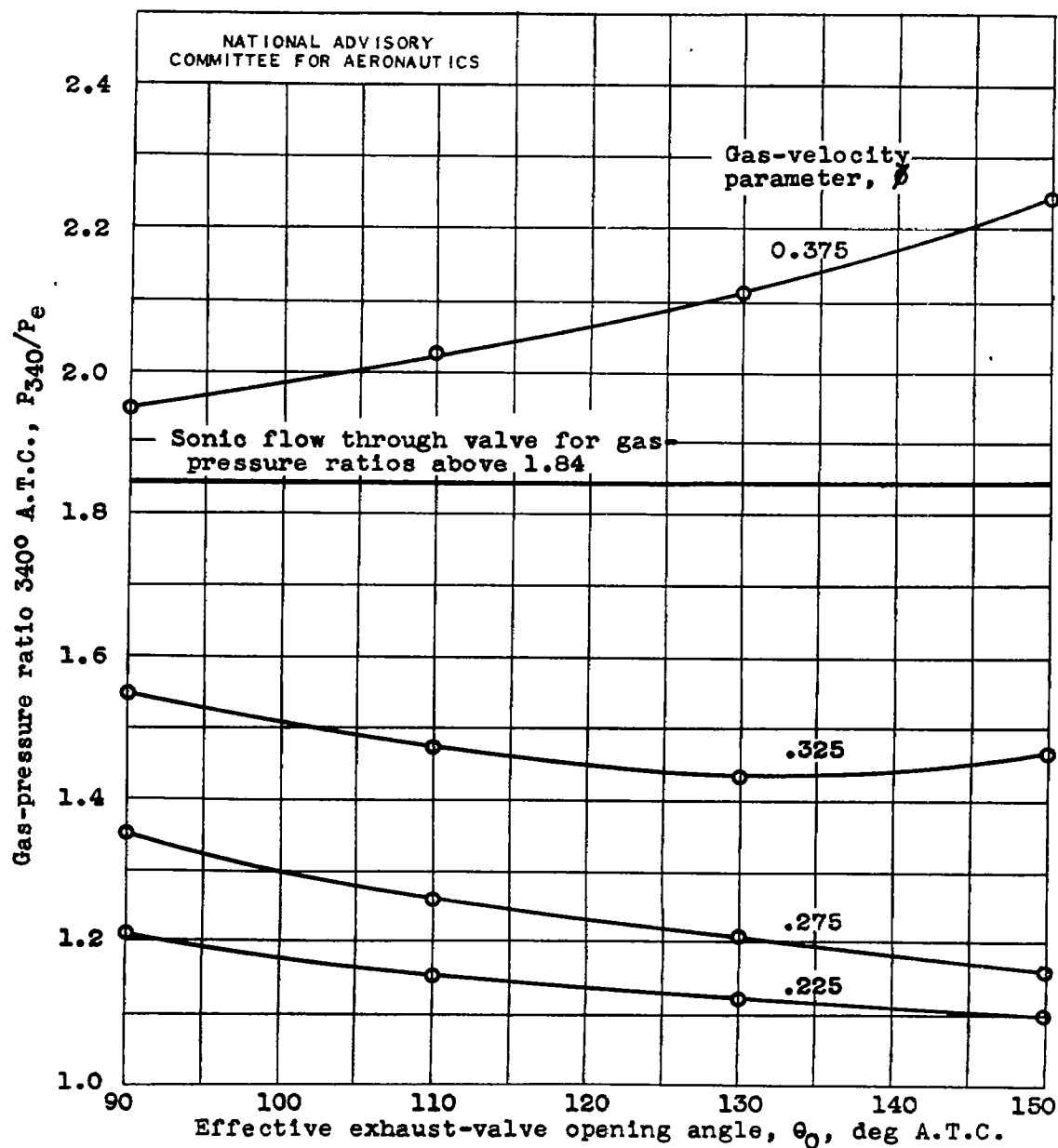


Figure 5. - Variation in gas-pressure ratio 340° A.T.C. with effective exhaust-valve opening angle for various values of gas-velocity parameter. Effective closing angle, 5° A.T.C.; compression ratio, 7.5 initial gas-pressure ratio has been varied with effective exhaust-valve opening angle according to adiabatic relation between pressure and volume, value being 9.33 at crank angle of 110° A.T.C.

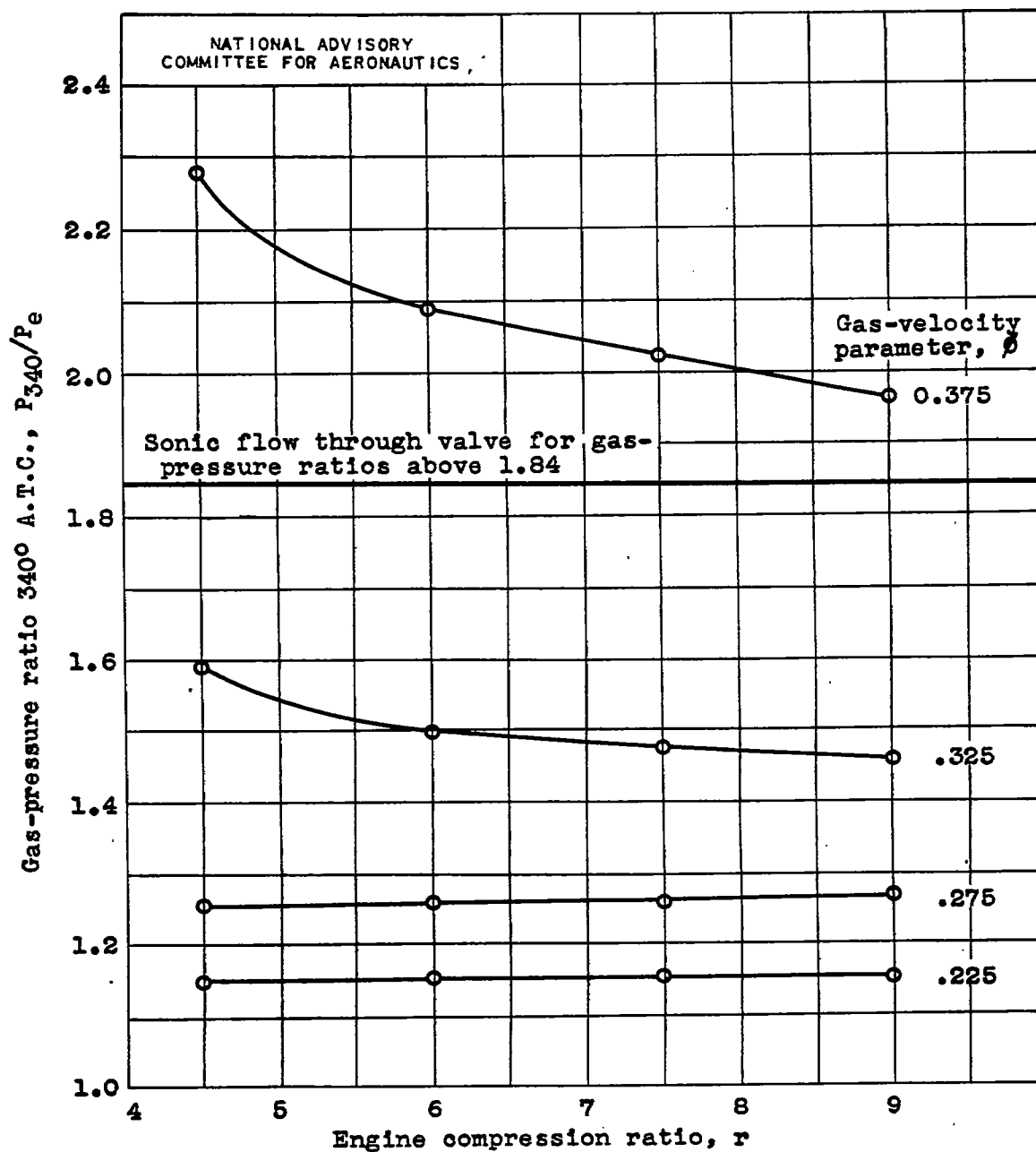


Figure 6. - Variation in gas-pressure ratio 340° A.T.C. with engine compression ratio for various values of gas-velocity parameter. Effective opening angle, 110° A.T.C.; effective closing angle, 5° A.T.C.; initial gas-pressure ratio, 9.33.

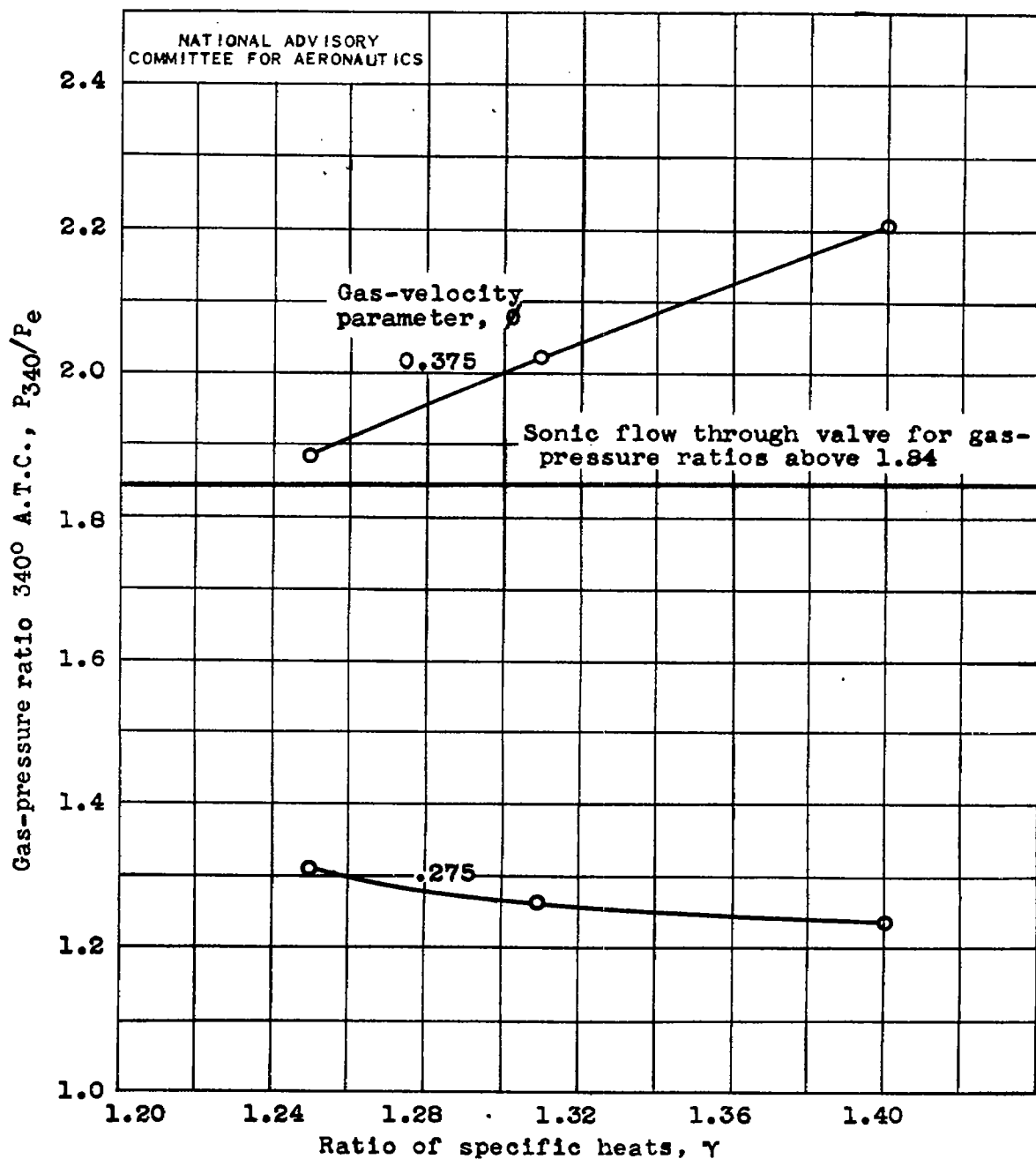


Figure 7. - Variation in gas-pressure ratio 340° A.T.C. with ratio of specific heats for two values of gas-velocity parameter. Effective opening angle, 110° A.T.C.; effective closing angle, 5° A.T.C.; initial gas-pressure ratio, 9.33; compression ratio, 7.5.

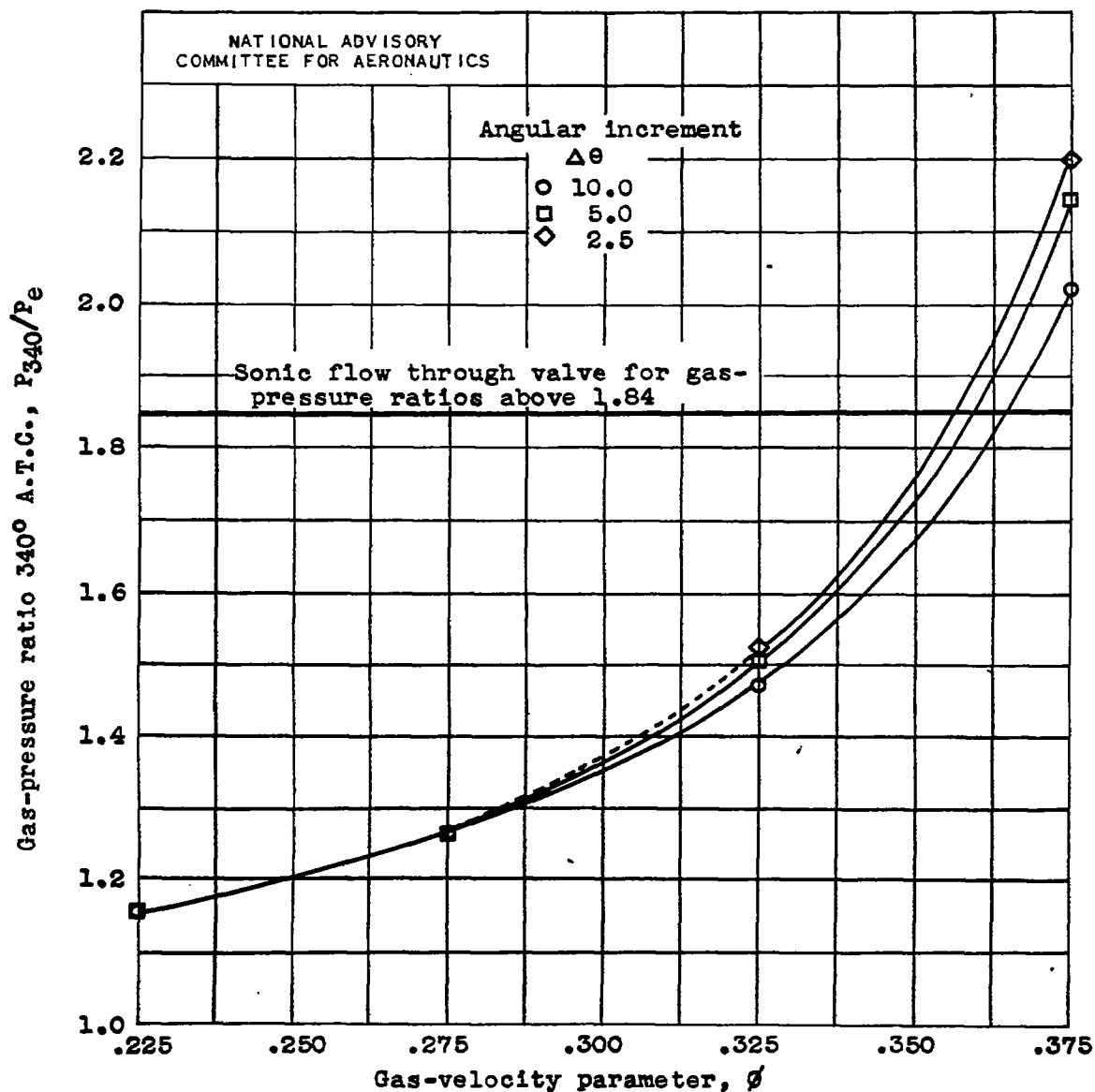


Figure 8. - Effect of angular increment used in point-by-point solution on gas-pressure ratio 340° A.T.C. Effective opening angle, 110° A.T.C.; effective closing angle, 5° A.T.C.; initial gas-pressure ratio, 9.33; compression ratio, 7.5.

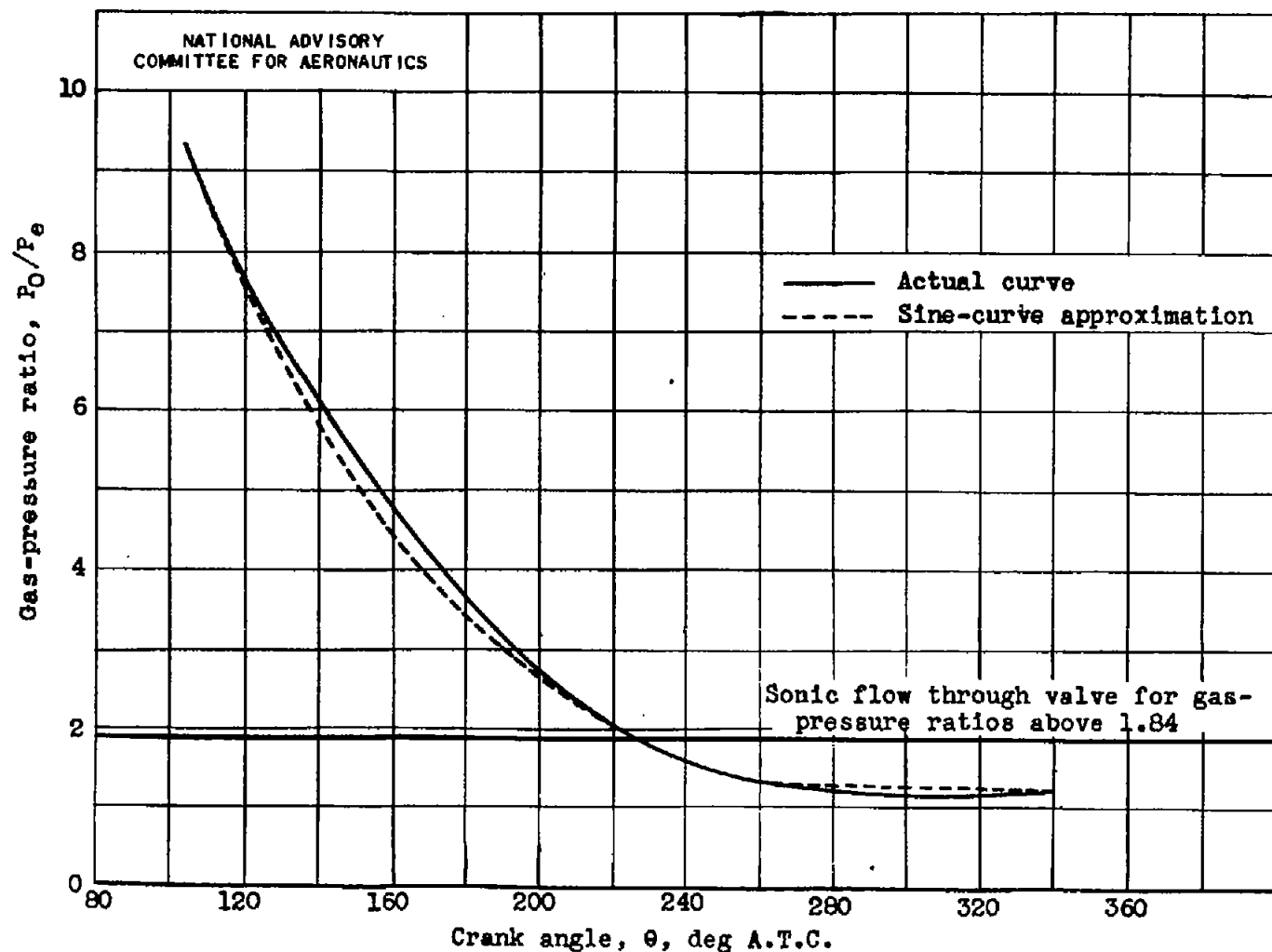


Figure 9. - Comparison of calculated gas-pressure ratios using actual and approximate exhaust-valve flow coefficients for engine A. Effective opening angle, 109° A.T.C.; effective closing angle, 21° A.T.C.; initial gas-pressure ratio 9.33 at the actual opening angle, 104° A.T.C.; compression ratio, 7.5; gas-velocity parameter, 0.297.